Understand Rational and Irrational Numbers

Think It Through

What are rational numbers?

Rational numbers are numbers that can be written as the quotient of two integers. Since the bar in a fraction represents division, every fraction whose numerator and denominator is an integer is a rational number.

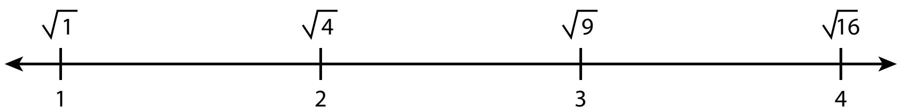
Any number that *could* be written as a fraction whose numerator and denominator are integers is also a rational number.

Think **Every integer, whole number, and natural number is a rational number.**

You can write every integer, whole number, and natural number as a fraction. So they are all rational numbers.

$3 = \frac{3}{1}$ $\quad -5 = -\frac{5}{1}$ $\quad 0 = \frac{0}{1}$ $\quad \sqrt{25} = 5 \text{ or } \frac{5}{1}$

The square root of a perfect square is also a rational number.



Think **Every terminating decimal is a rational number.**

You can write every terminating decimal as a fraction. They are all rational numbers.

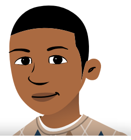
You can use what you know about place value to find the fraction that is equivalent to any terminating decimal.

0.4	*four tenths*	$\frac{4}{10} = \frac{2}{5}$
0.75	*seventy-five hundredths*	$\frac{75}{100} = \frac{3}{4}$
0.386	*three hundred eighty-six thousandths*	$\frac{386}{1,000} = \frac{193}{500}$
$\sqrt{0.16} = 0.4$	*four tenths*	$\frac{4}{10} = \frac{2}{5}$

Think Every repeating decimal is a rational number.

You can write every repeating decimal as a fraction.
So repeating decimals are all rational numbers.

As an example, look at the repeating decimal $0.\overline{3}$.



Let $x = 0.\overline{3}$

$10 \cdot x = 10 \cdot 0.\overline{3}$

$10x = 3.\overline{3}$

The repeating portion goes to the tenths place. Multiply both sides by 10.

$10x - x = 3.\overline{3} - 0.\overline{3}$

Subtract x from the left side and $0.\overline{3}$ from the right side. The equation is still balanced because x and $0.\overline{3}$ are equal.

$9x = 3$

$\frac{9x}{9} = \frac{3}{9}$

$x = \frac{3}{9}$ or $\frac{1}{3}$

$0.\overline{3} = \frac{1}{3}$

Here's another example of how you can write a repeating decimal as a fraction.

$x = 0.\overline{512}$

$1{,}000x = 512.\overline{512}$

The repeating portion goes to the thousandths place. Multiply by 1,000.

$1{,}000x - x = 512.\overline{512} - 0.\overline{512}$

Subtract x from the left side and **the repeating decimal** from the right side.

$999x = 512$

$x = \frac{512}{999}$

Reflect

1 What fraction is equivalent to 5.1? Is 5.1 a rational number? Explain.

Think About Estimating Irrational Numbers

Let's Explore the Idea **What numbers are not rational? Let's look at a number like $\sqrt{2}$, the square root of a number that is not a perfect square.**

2 Look at the number line below. The number $\sqrt{2}$ is between $\sqrt{1}$ and $\sqrt{4}$. Since $\sqrt{1} = 1$ and $\sqrt{4} = 2$, that means that $\sqrt{2}$ must be between what two integers?

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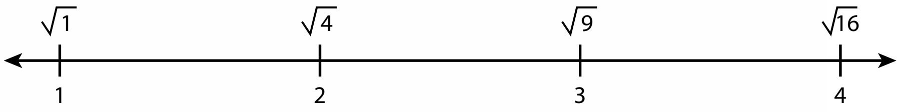
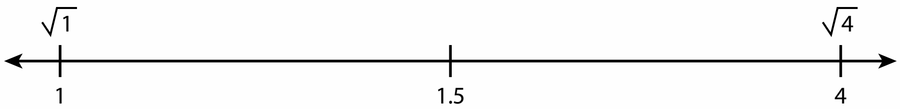


3 Draw a point on the number line where you would locate $\sqrt{2}$. Where did you draw the point? _______________

4 Calculate: $1.3^2 =$ ______ $1.4^2 =$ ______ $1.5^2 =$ ______

5 Based on your calculations, draw a point on the number line below where you would locate $\sqrt{2}$ now. Where did you draw the point? _______________



6 Calculate: $1.41^2 =$ ________ $1.42^2 =$ ________

7 Based on these calculations, $\sqrt{2}$ is between which two decimals? _______________

8 You can continue to estimate, getting closer and closer to the value of $\sqrt{2}$. For example, $1.414^2 = 1.999396$ and $1.415^2 = 2.002225$, but you will never find a terminating decimal that multiplied by itself equals 2. The decimal will also never have a repeating pattern.

$\sqrt{2}$ cannot be expressed as a terminating or repeating decimal, so it cannot be written as a fraction. Numbers like $\sqrt{2}$ and $\sqrt{5}$ are not rational. You can only estimate their values. They are called **irrational numbers.** Here, *irrational* means "cannot be set as a ratio." The set of rational and irrational numbers together make up the set of **real numbers.**

Now try this problem.

9 The value π is a decimal that does not repeat and does not terminate. Is it a rational or irrational number? Explain.

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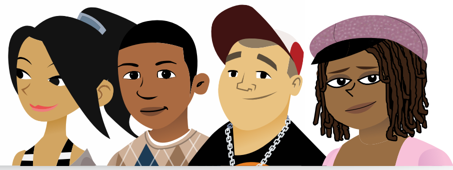
Let's Talk About It **You can estimate the value of an irrational number like $\sqrt{5}$ and locate that value on a number line.**



10 $\sqrt{5}$ is between which two integers? Explain your reasoning.

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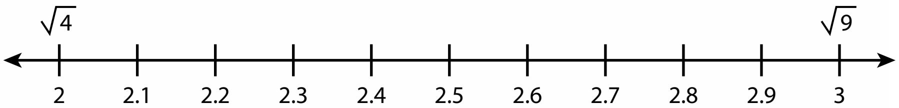
11 Mark a point at an approximate location for $\sqrt{5}$ on the number line below. $\sqrt{5}$ is between which two decimals to the tenths place? ____________



12 Calculate: $2.22^2 =$ ________ $2.23^2 =$ ________ $2.24^2 =$ ________

Based on your results, $\sqrt{5}$ is between which two decimals to the hundredths place?

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13 Draw a number line from 2.2 to 2.3. Label tick marks at hundredths to show 2.21, 2.22, 2.23, and so on. Mark a point at the approximate location of $\sqrt{5}$ to the thousandths place.

Try It Another Way **Explore using a calculator to estimate irrational numbers.**

14 Enter $\sqrt{5}$ on a calculator and press Enter. What is the result on your screen?

____

15 If this number is equal to $\sqrt{5}$, then the number squared should equal ________.

16 Clear your calculator. Then enter your result from problem 14. Square the number. What is the result on your screen? ______________________________

17 Explain this result. ______________________________

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Connect Estimating Irrational Numbers

Talk through these problems as a class, then write your answers below.

18 **Illustrate** Show that $0.\overline{74}$ is equivalent to a fraction. Is $0.\overline{74}$ a rational or irrational number? Explain.

19 **Analyze** A circle has a circumference of 3π inches. Is it possible to state the exact length of the circumference as a decimal? Explain.

20 **Create** Draw a Venn diagram showing the relationships among the following sets of numbers: integers, irrational numbers, natural numbers, rational numbers, real numbers, and whole numbers.

Apply Estimating Irrational Numbers

Use what you have learned to complete this task.

21 Put It Together Consider these numbers:

$\sqrt{50}$ $3.4\overline{56}$ 0 $\sqrt{\frac{4}{9}}$ 0.38 $\sqrt{81}$ 2π $\sqrt{1.69}$ $\sqrt{\frac{2}{9}}$

Part A Write each of the numbers in the list above in the correct box.

Rational Numbers	Irrational Numbers

Part B Circle one of the numbers you said was rational. Explain how you decided that the number was rational.

Part C Now circle one of the numbers you said was irrational. Explain how you decided that the number was irrational.

Part D Draw a number line and locate the two numbers you circled on the line. Write a comparison statement using <, =, or > to compare the numbers.